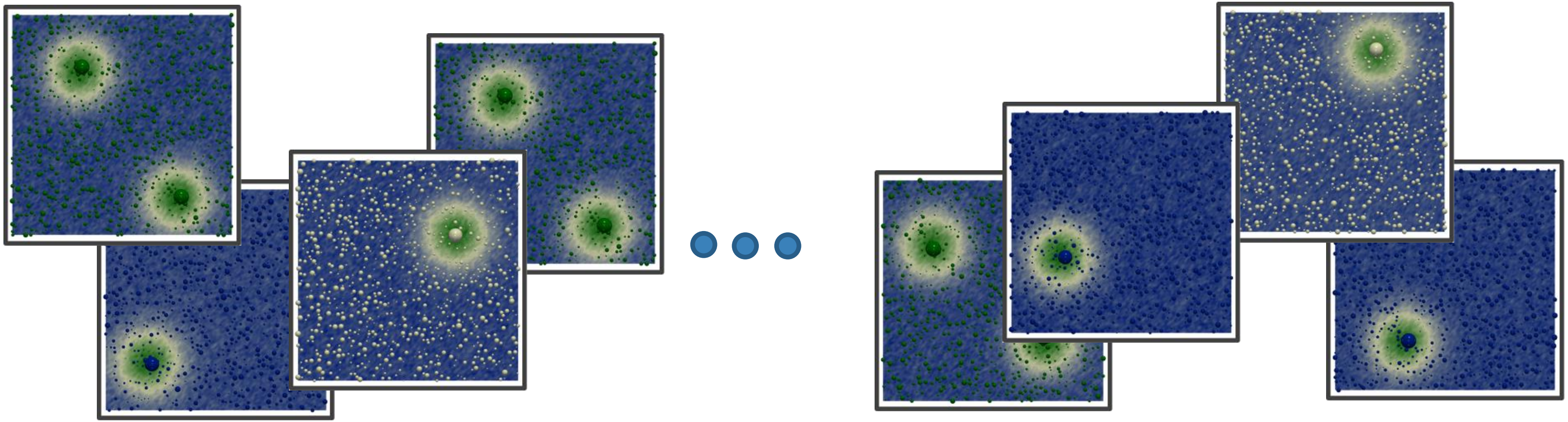
Persistence Atlas for Critical Point Variability in Ensembles

Guillaume Favelier, **Noura Faraj**, Brian Summa, and Julien Tierny

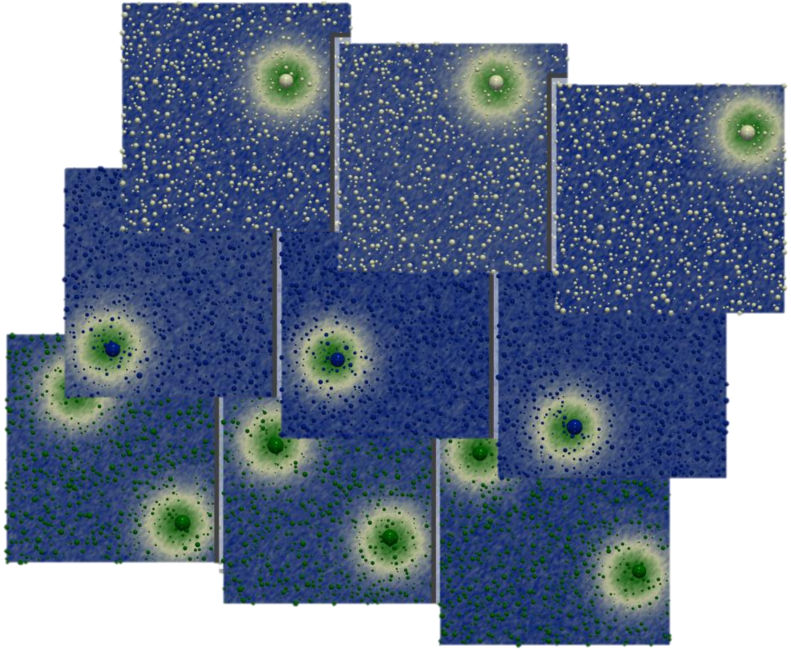
Ensemble datasets

- Collection of piecewise linear (PL) scalar fields $f : \mathcal{M} \rightarrow \mathbb{R}$
- Challenging to analyze, visualize and interpret



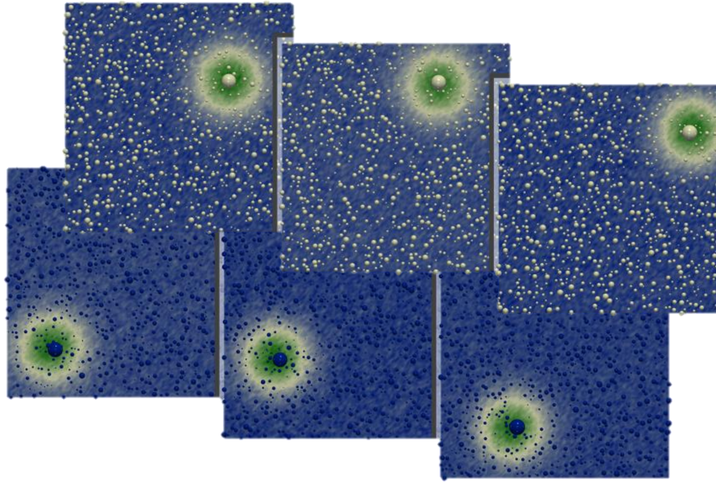
Ensemble datasets

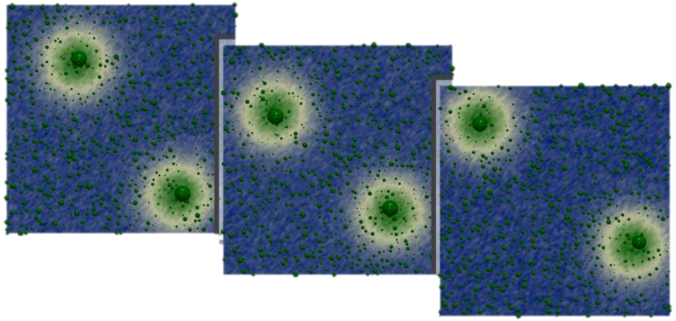
- Analyze features of interest



Ensemble datasets

- Analyze features of interest
  - Trend variability

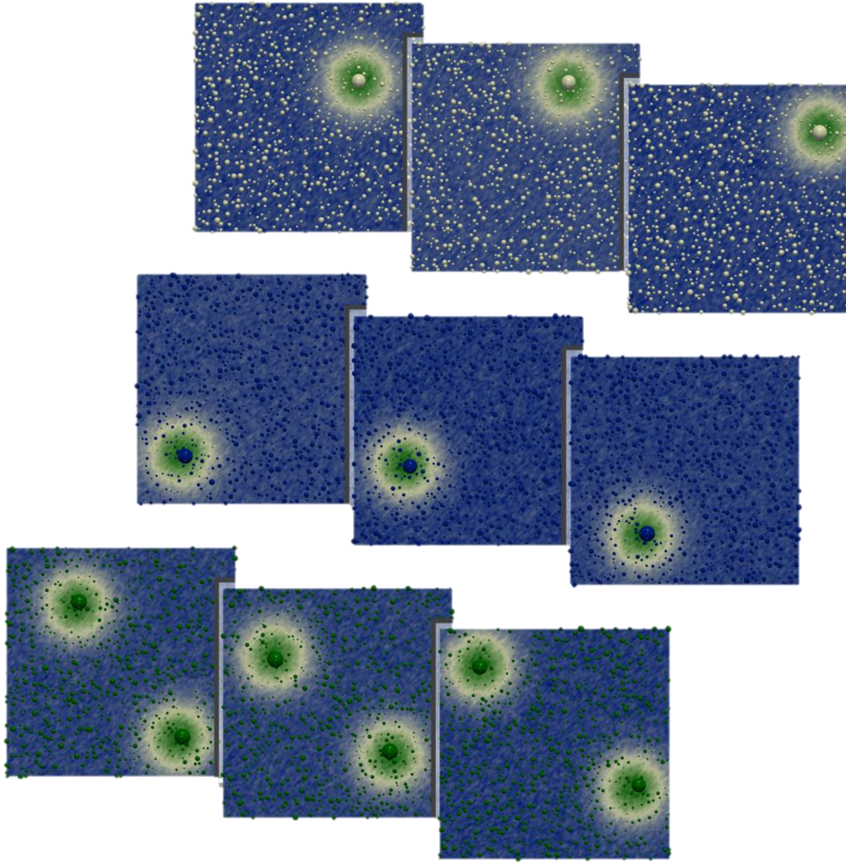




Ensemble datasets

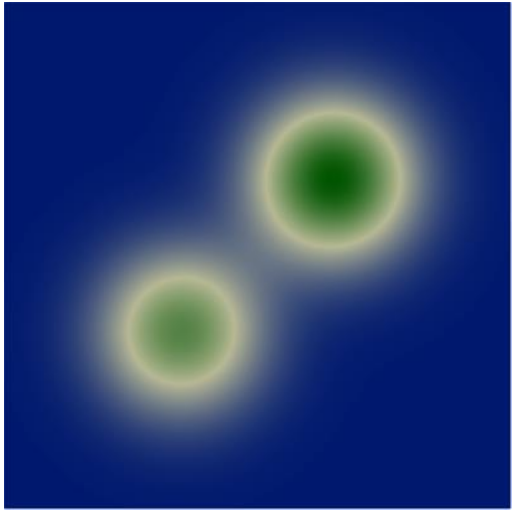
- Analyze features of interest
  - Trend variability
  - Spatial variability

→Topological data analysis



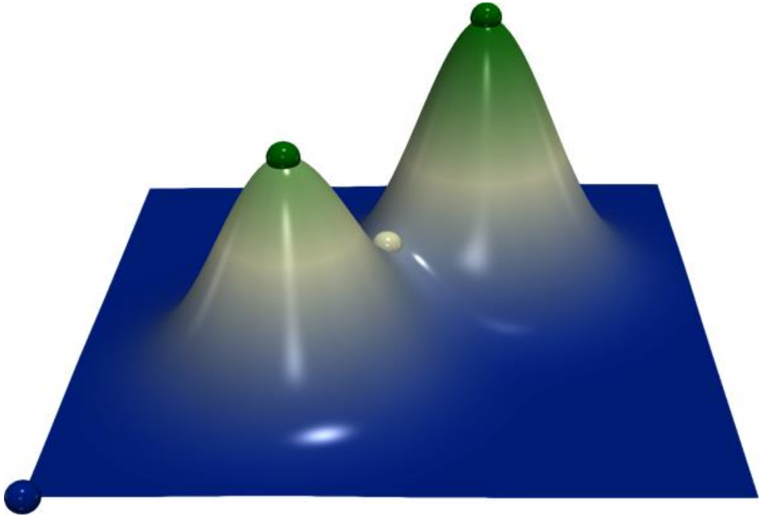
Topological data analysis

- PL scalar field $f : \mathcal{M} \to \mathbb{R}$



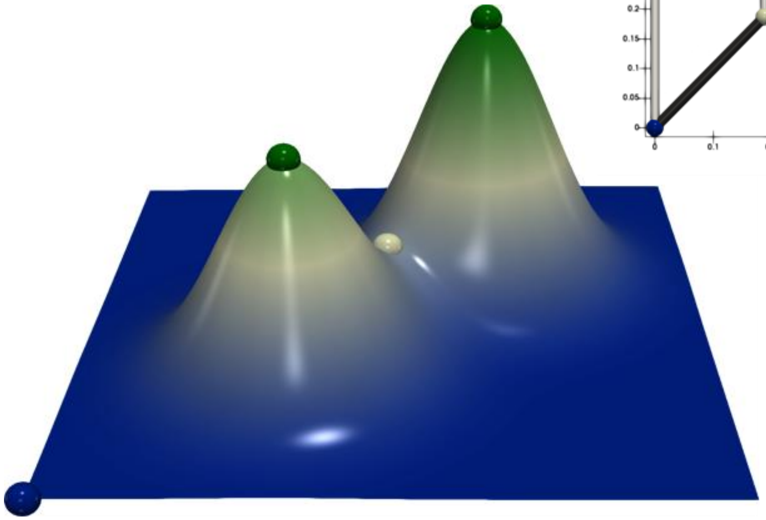
Topological data analysis

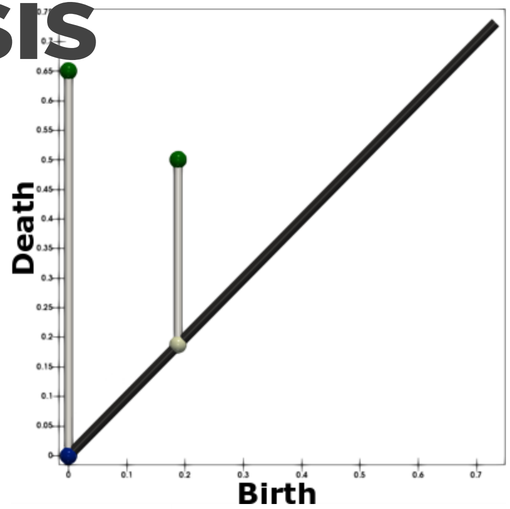
- PL scalar field $f : \mathcal{M} \rightarrow \mathbb{R}$
- Topological abstractions
  - Critical points

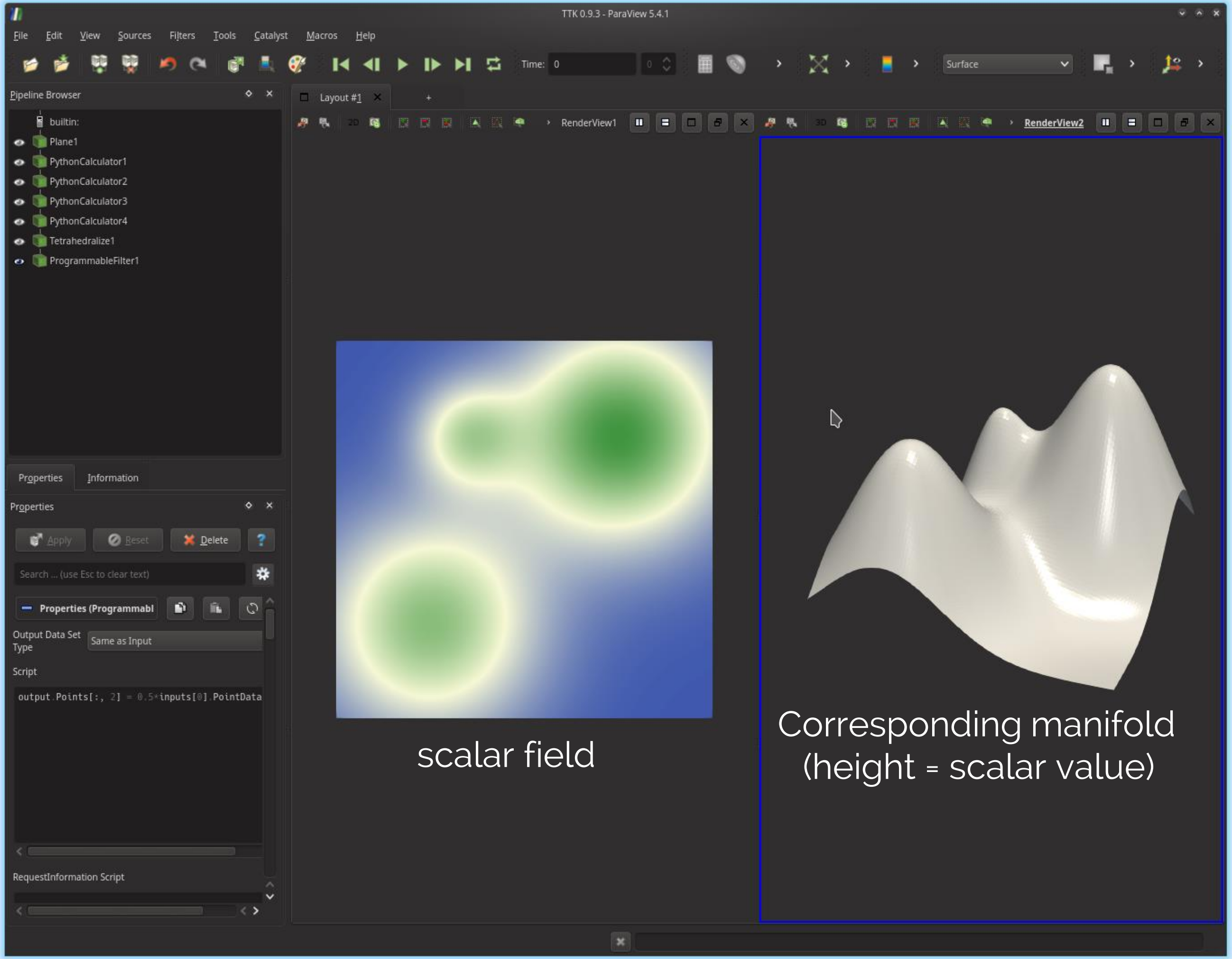


Topological data analysis

- PL scalar field $f : \mathcal{M} \rightarrow \mathbb{R}$
- Topological abstractions
  - Critical points
  - Persistence diagrams






TTK 0.9.3 - ParaView 5.4.1
File
Edit
View
Sources
Filters
Tools
Catalyst
Macros
Help
Time: 0
Surface
Pipeline Browser
Layout #1
RenderView1
RenderView2
builtin:
Plane1
PythonCalculator1
PythonCalculator2
PythonCalculator3
PythonCalculator4
Tetrahedralize1
ProgrammableFilter1
Properties
Information
Properties
Apply
Reset
Delete
Search ... (use Esc to clear text)
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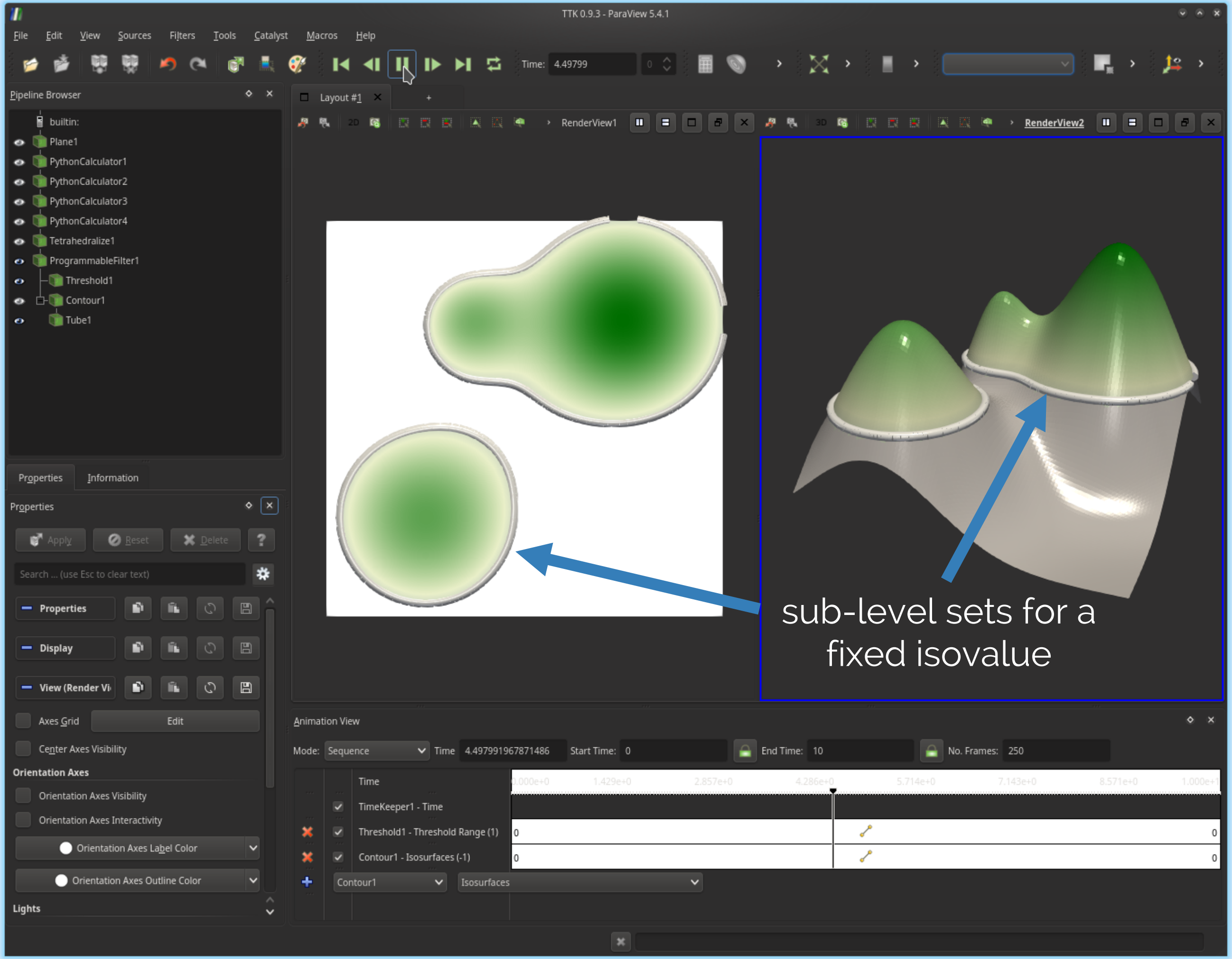
Output Data Set Type
Same as Input
Script
output.Points[:, 2] = 0.5*inputs[0].PointData
RequestInformation Script
scalar field
Corresponding manifold
(height = scalar value)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File Edit View Sources Filters Tools Catalyst Macros Help
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Pipeline Browser
builtin:
Plane1
PythonCalculator1
PythonCalculator2
PythonCalculator3
PythonCalculator4
Tetrahedralize1
ProgrammableFilter1
Threshold1
Contour1
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Layout #1
RenderView1
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Orientation Axes Interactivity
Orientation Axes Label Color
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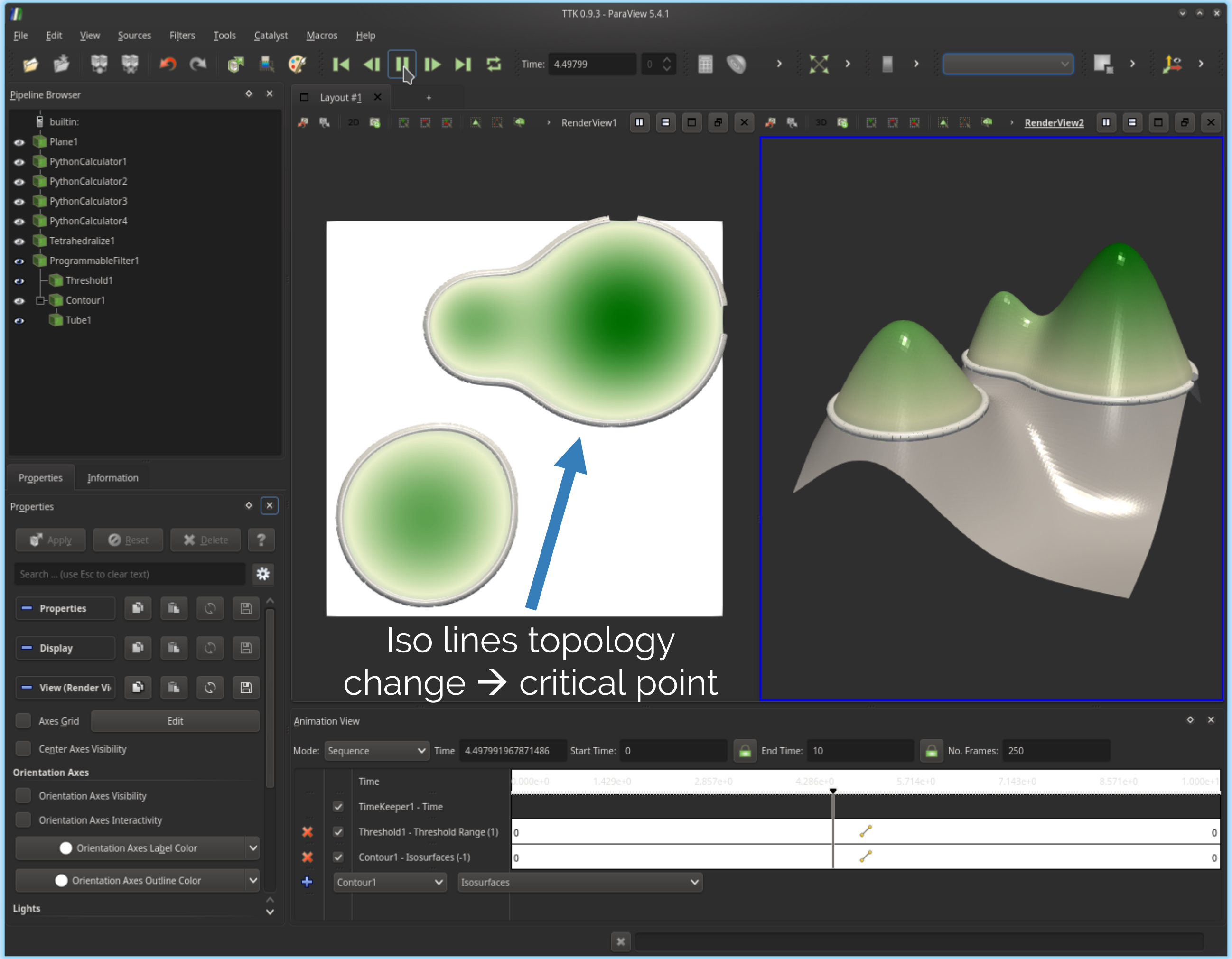
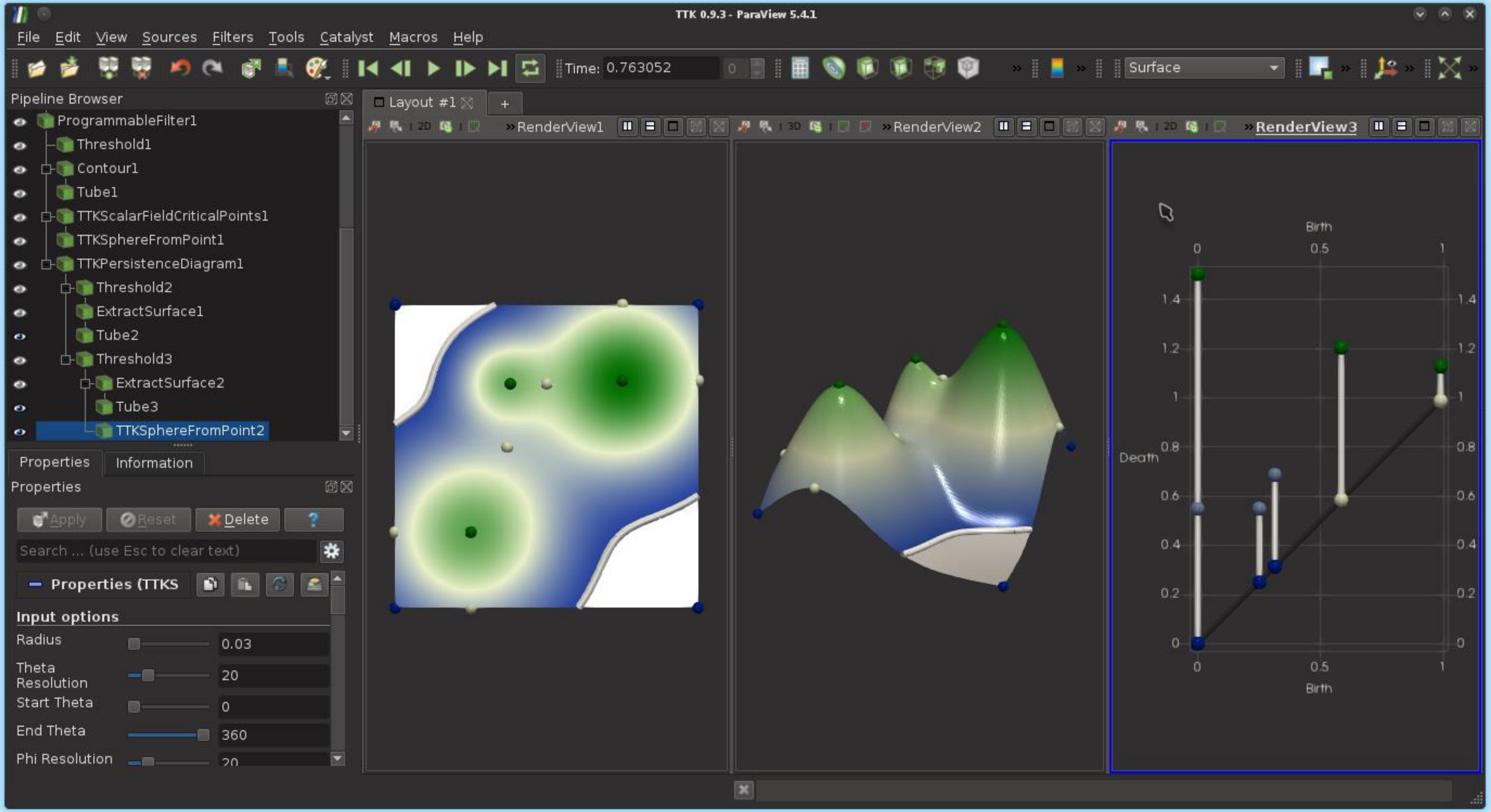
Animation View
Mode: Sequence
Time 4.497991967871486
Start Time: 0
End Time: 10
No. Frames: 250
Time
TimeKeeper1 - Time
Threshold1 - Threshold Range (1)
Contour1 - Isosurfaces (-1)
Contour1
Isosurfaces
sub-level sets for a
fixed isovalue



Iso lines topology change → critical point

Increase isovalue



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File Edit View Sources Filters Tools Catalyst Macros Help
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Surface
Pipeline Browser
ProgrammableFilter1
Threshold1
Contour1
Tube1
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TTKSphereFromPoint1
TTKPersistenceDiagram1
Threshold2
ExtractSurface1
Tube2
Threshold3
ExtractSurface2
Tube3
TTKSphereFromPoint2
Properties
Information
Apply
Reset
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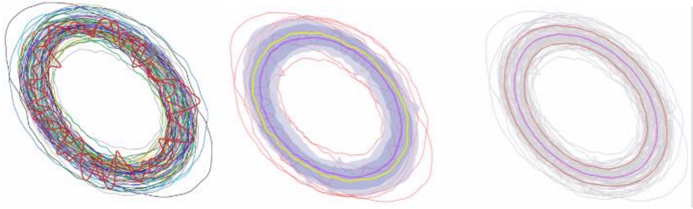
Search ... (use Esc to clear text)
Properties (TTKS
Input options
Radius 0.03
Theta Resolution 20
Start Theta 0
End Theta 360
Phi Resolution 20
Layout #1
RenderView1
RenderView2
RenderView3
Birth
Death

Contributions

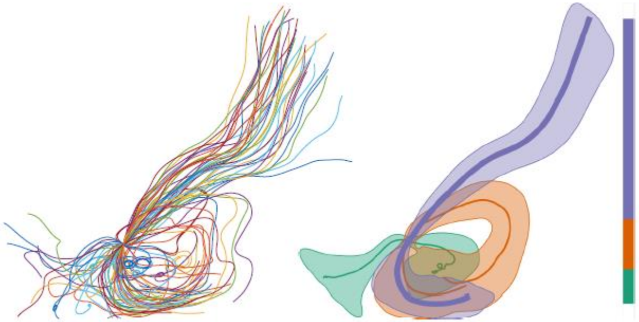
- Informative representation of critical point layout
- Statistical space for critical point layout
- Display of confidence regions per cluster for critical points layouts
- Implementation
  - VTK-based C++ implementation (additional material)

Previous work

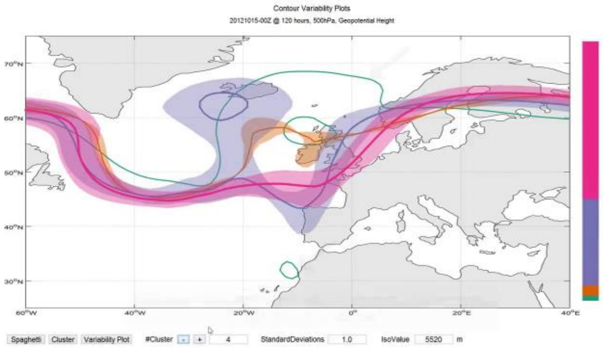
- Level sets
- Stream lines



[Whitaker et al 2013]

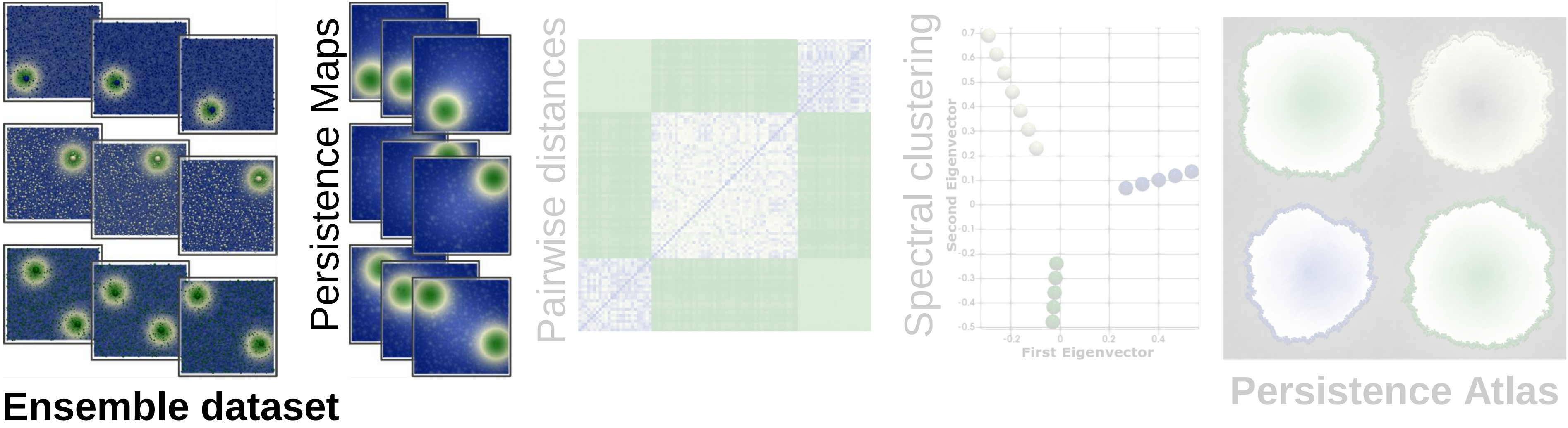


[Ferstl et al 2016]



[Ferstl et al 2016]

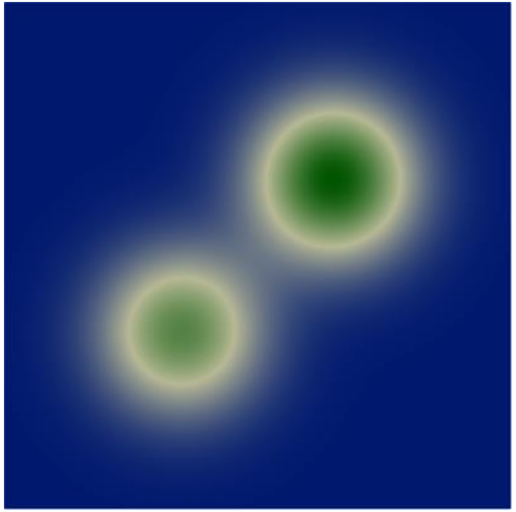
Overview

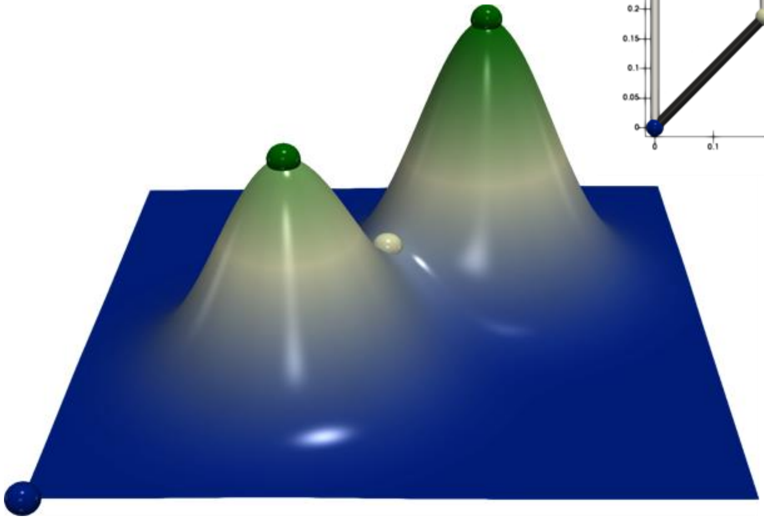


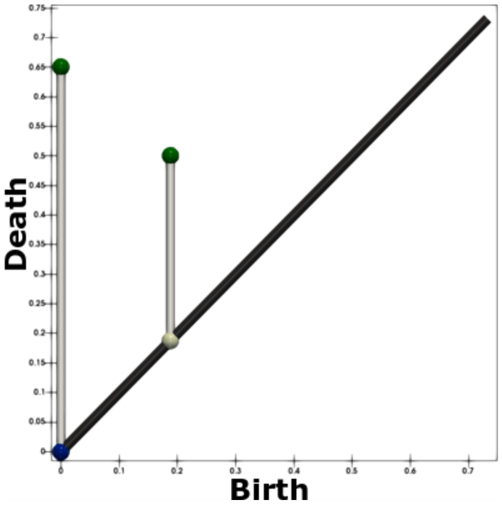
Persistence Maps
Pairwise distances
Spectral clustering
Second Eigenvector
First Eigenvector
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
-0.1
-0.2
-0.3
-0.4
-0.5
-0.2
0
0.2
0.4
Ensemble dataset
Persistence Atlas

Ideal input data

- PL scalar field $f : \mathcal{M} \to \mathbb{R}$

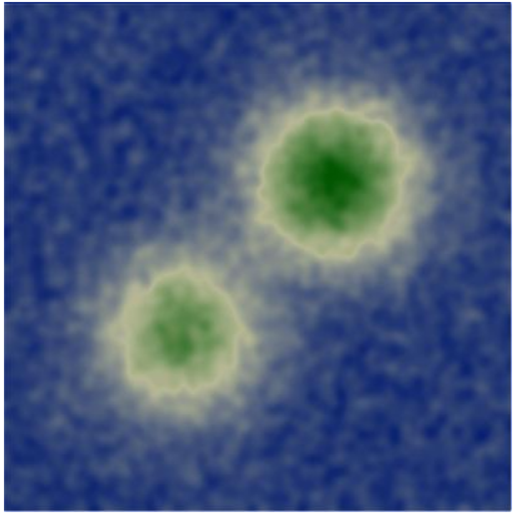


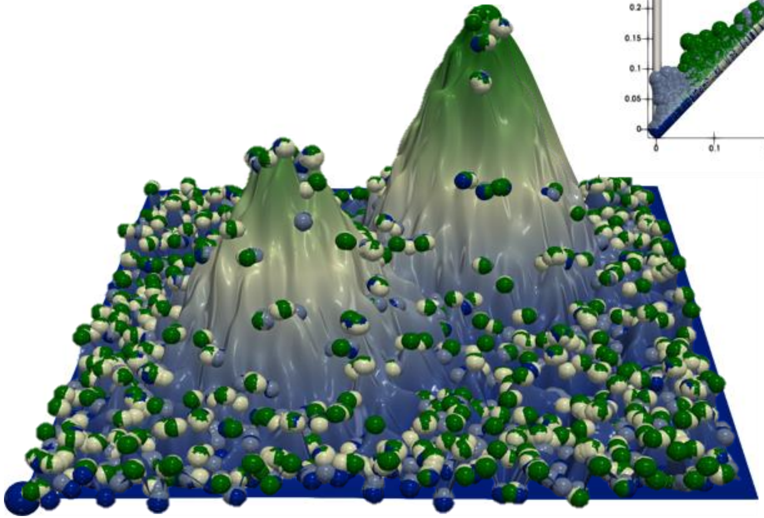


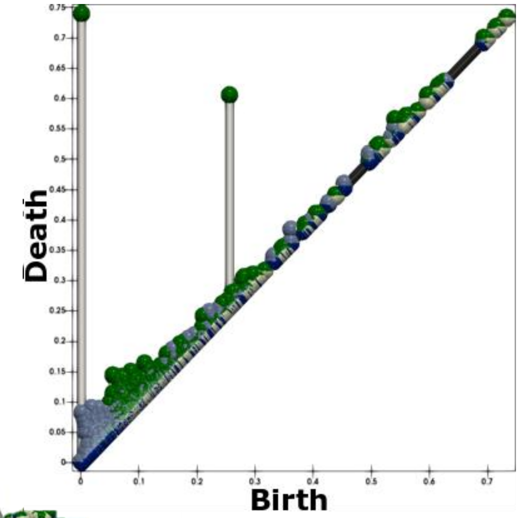


Actual input data

- PL scalar field $f : \mathcal{M} \to \mathbb{R}$







Persistence Map

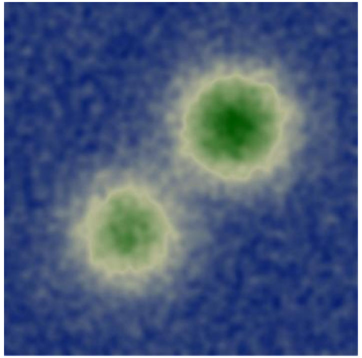
- Transformation weighted by topological persistence
- Sum of radial basis functions

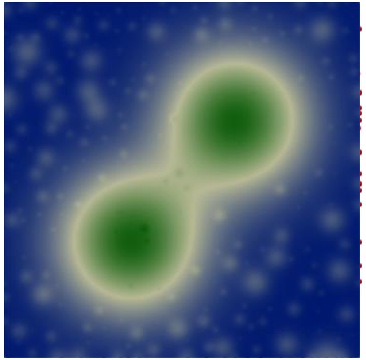
$$\phi(v) = \sum_{c \in C} \alpha(c) e^{-\frac{\|v-c\|_2^2}{2\sigma(c)^2}} \qquad \alpha(c) = P(c), \quad \sigma(c) = \gamma P(c)$$

Local critical point density measure

Persistence

→ Highlights salient features





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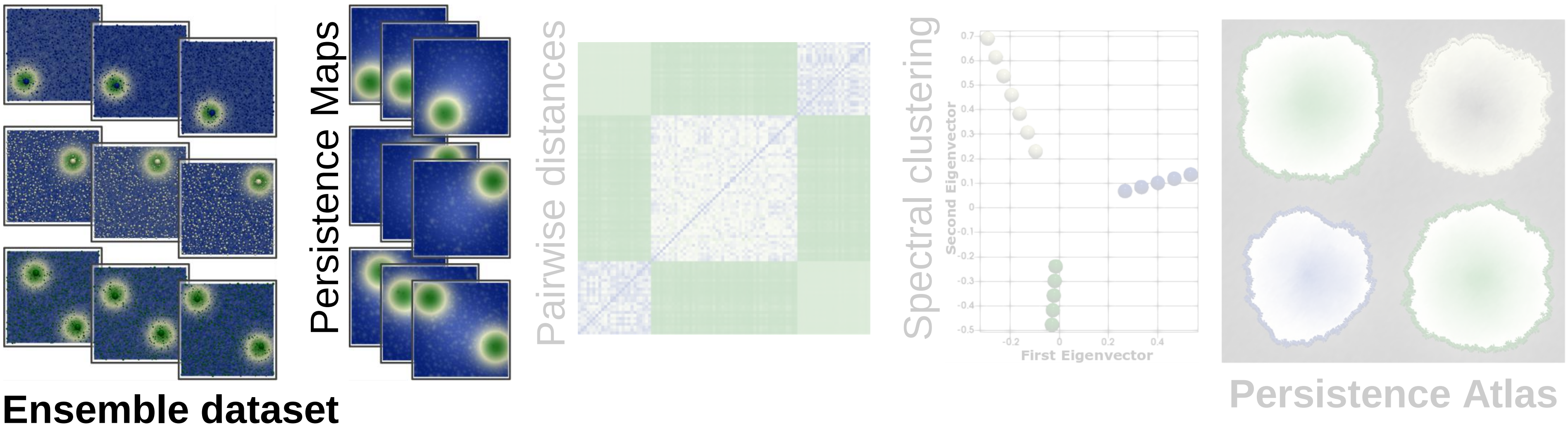

Persistence Maps
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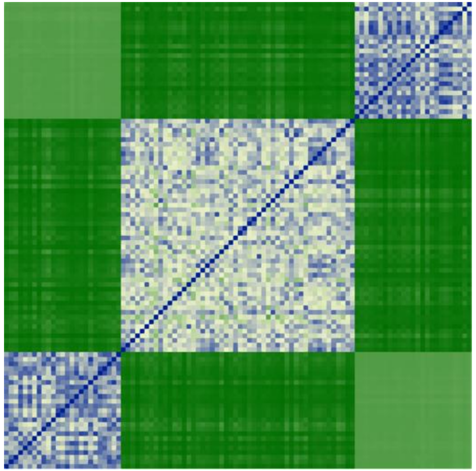
First Eigenvector
Ensemble dataset
Persistence Atlas

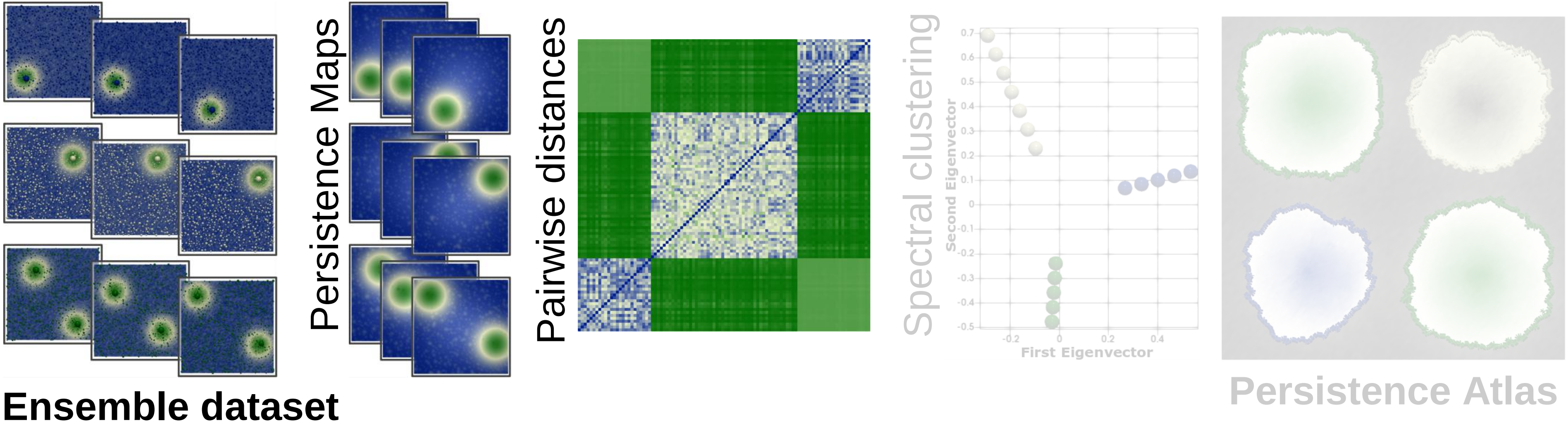
Members comparison

- Distance matrix
  - Pairwise distance between persistence maps
  - Evaluation using L_2 norm



→ Used to compare spatial layout of critical points between members

Overview



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Persistence Atlas

Persistence maps clustering

1 - Low dimensional embedding
2 - Clustering using K-means

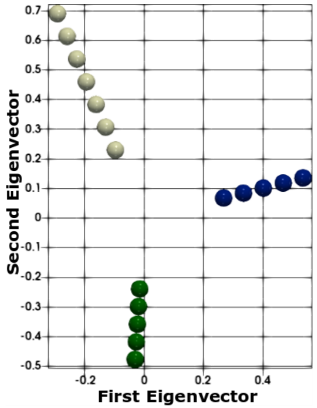
- Automated detection of
  - Number of clusters
  - Embedding dimension

Spectral embedding

Laplacian eigenmaps [Belkin et al 2003]

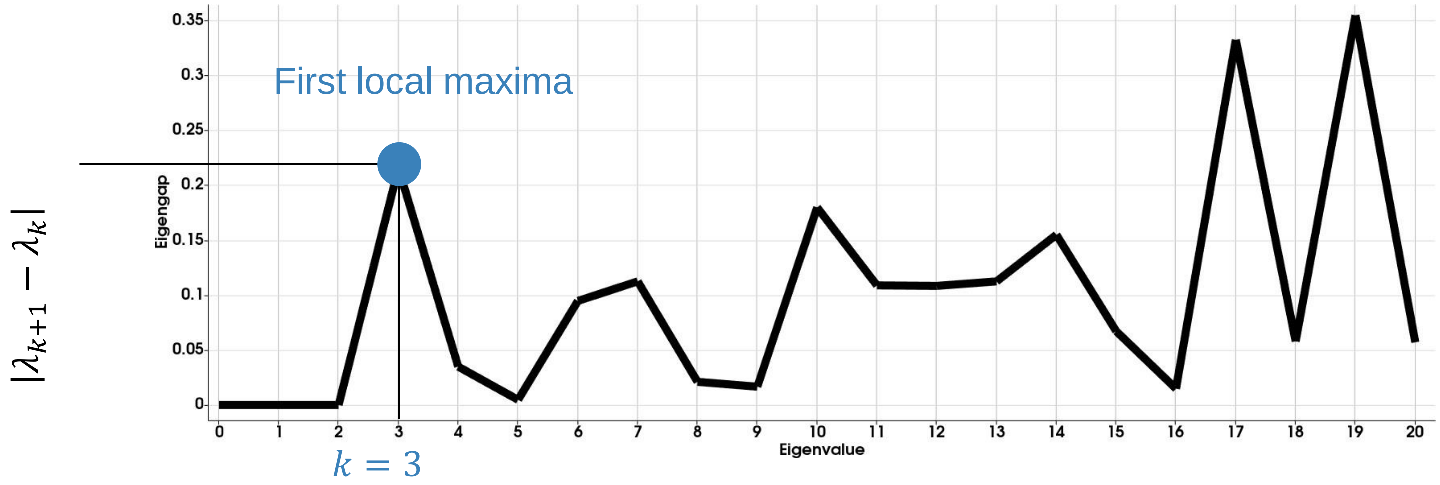
- Adjacency graph
  - n nearset neighbors (from distance matrix)
- Laplacian of the graph
  - Sparse matrix semidefinite positive
- Projecting each member along the n_d first eigenvectors
- First two components used for visual assessment of the members repartitions

→ Quantitative assessement: clustering



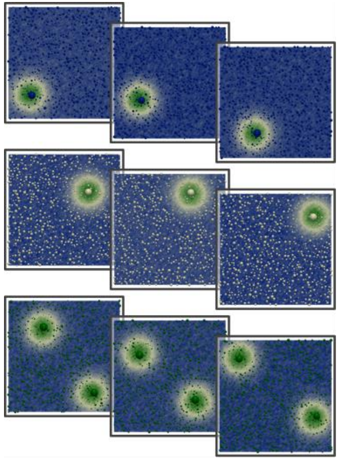
Spectral clustering

- K-means, number of cluster k ?
- Suggested by looking at eigengaps [Luxburg 2007]



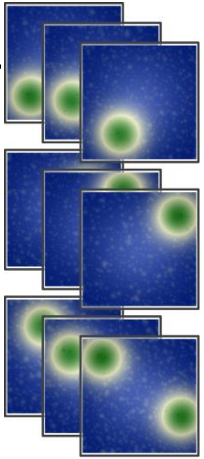
Note: embedding dimension $n_d = k - 1$

Overview

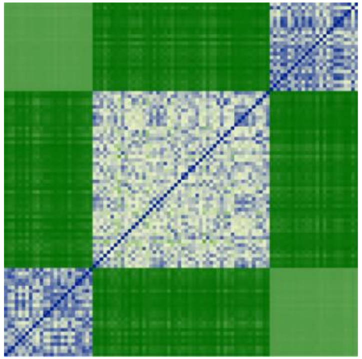


Ensemble dataset

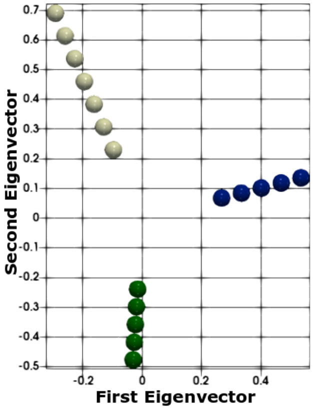
Persistence Maps



Pairwise distances



Spectral clustering

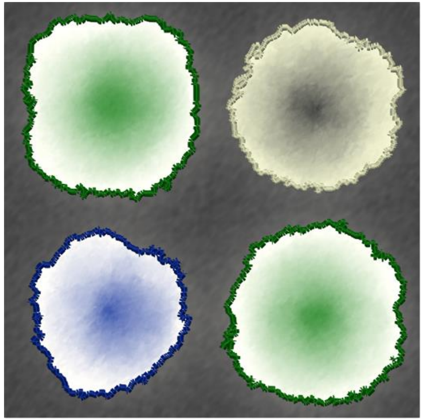


Persistence Atlas

Cluster analysis

- Collection of critical points per cluster
  - **spatial variability** of the common topological structure

→ Visualization using mandatory critical points [Guenther et al 2014]
  - Positional uncertainty of critical points
  - Point-wise intervals
  - Well suited for analysis of ensemble data



Mandatory critical points [Guenther 2014]

- Separation between minima and maxima
- Area where at least one critical point must occur

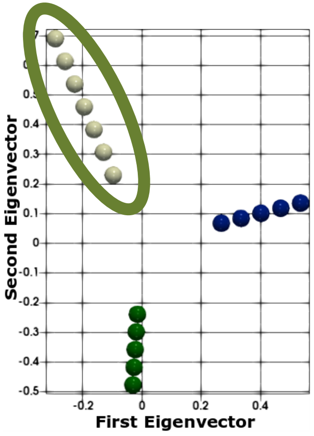
→ Provides a set of connected components in which critical points will be found

Mandatory critical points

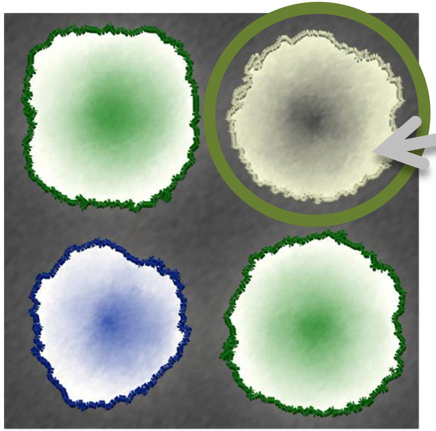
[Guenther 2014]

- Visualization

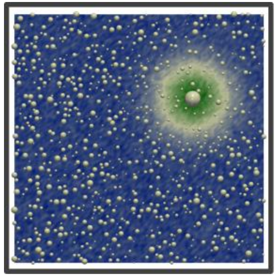
3 clusters



Critical component



All members of cluster 1 have a maxima in this region

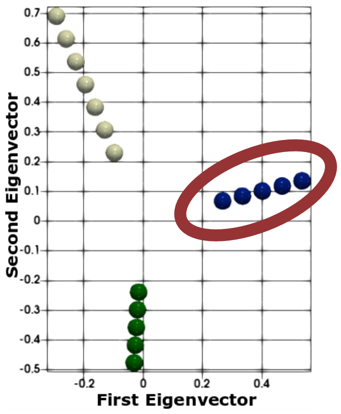


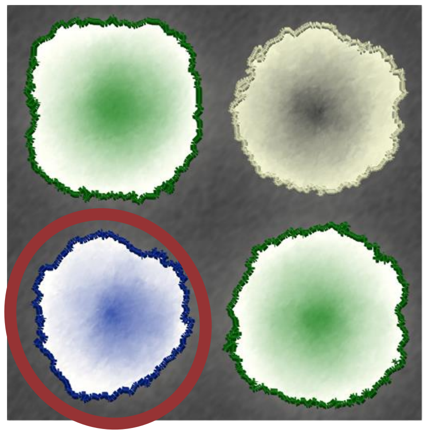
Mandatory critical points

[Guenther 2014]

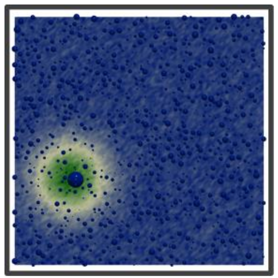
- Visualization

3 clusters





Critical component

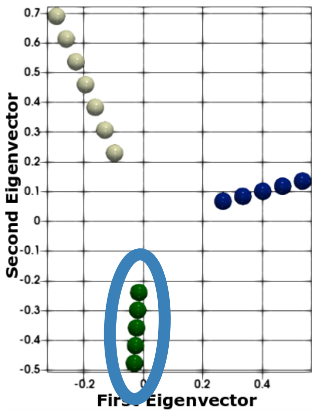


Mandatory critical points

[Guenther 2014]

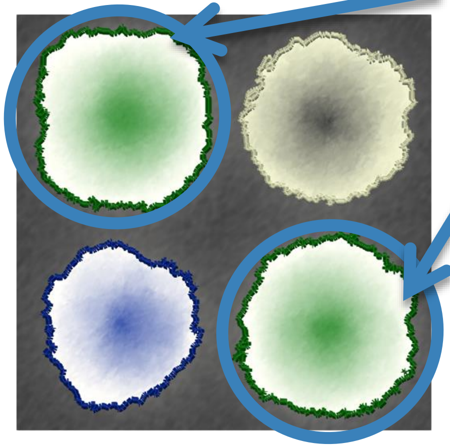
- Visualization

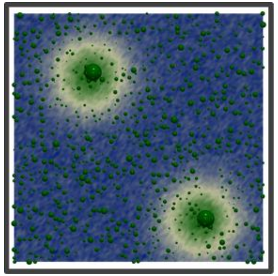
3 clusters



All members of cluster 2 have a maxima in **EACH** component

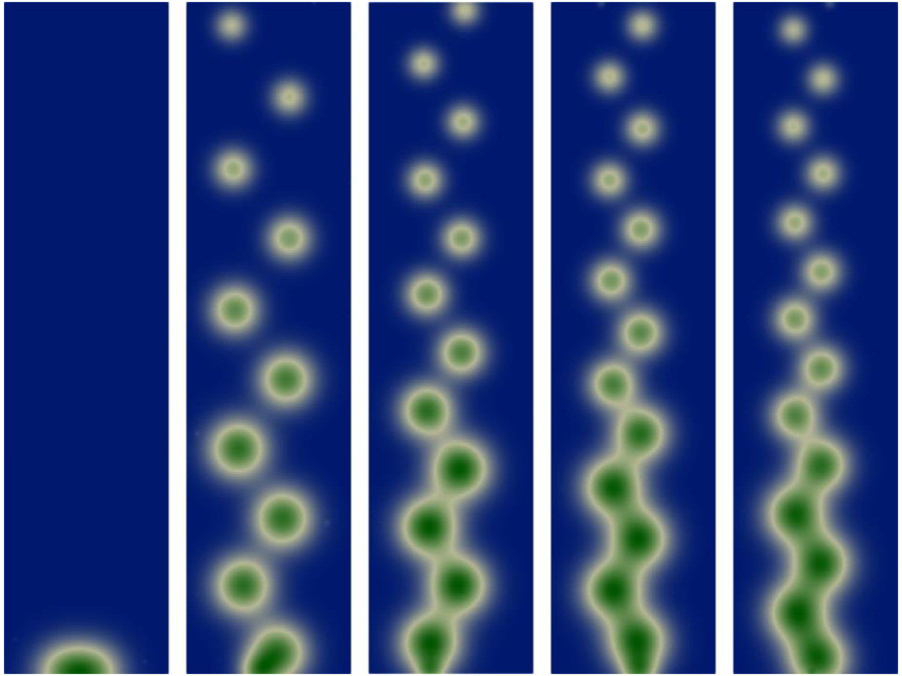
Critical components





Results

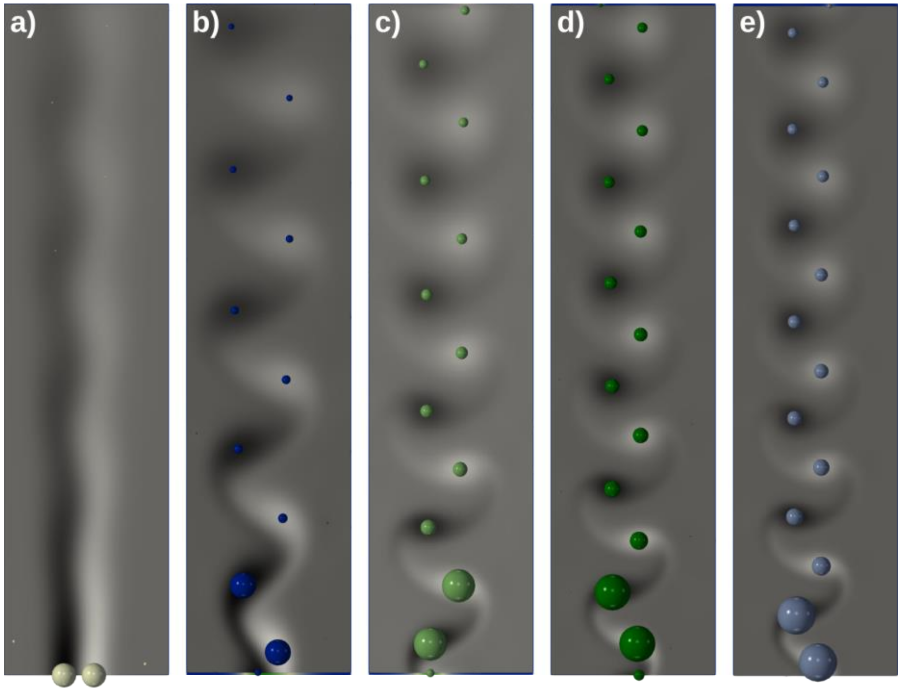
Vortex street ensemble



Persistence maps

- 45 von Karman vortex streets
- Scalar data: orthogonal component of the curl

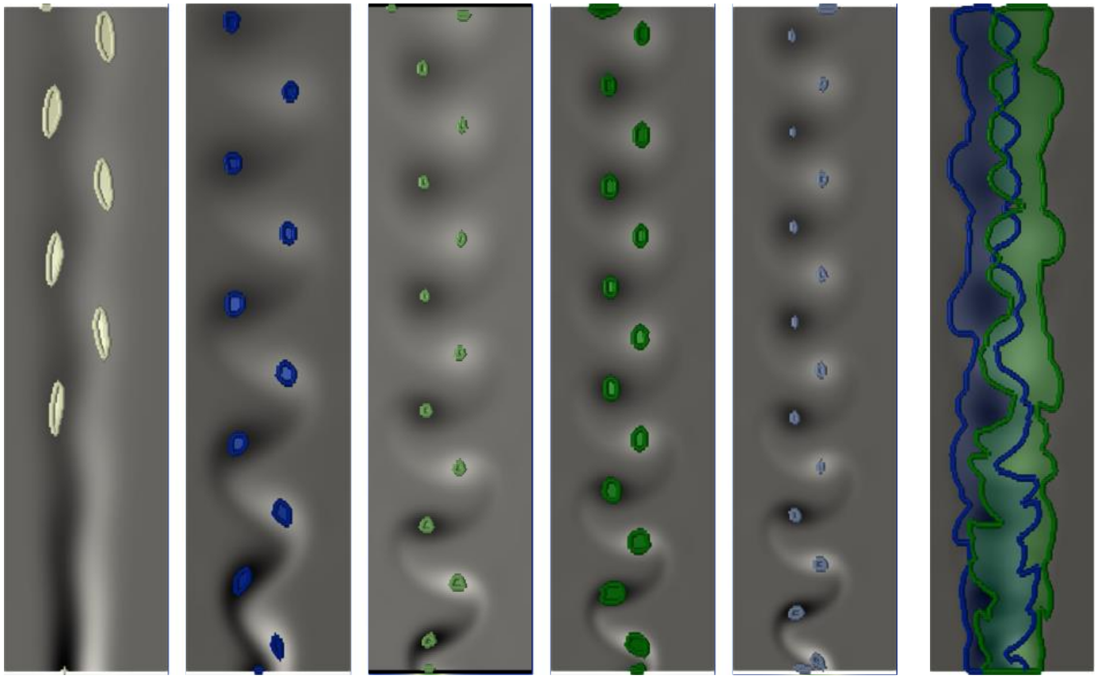
Vortex street ensemble



Critical points: vortices centers

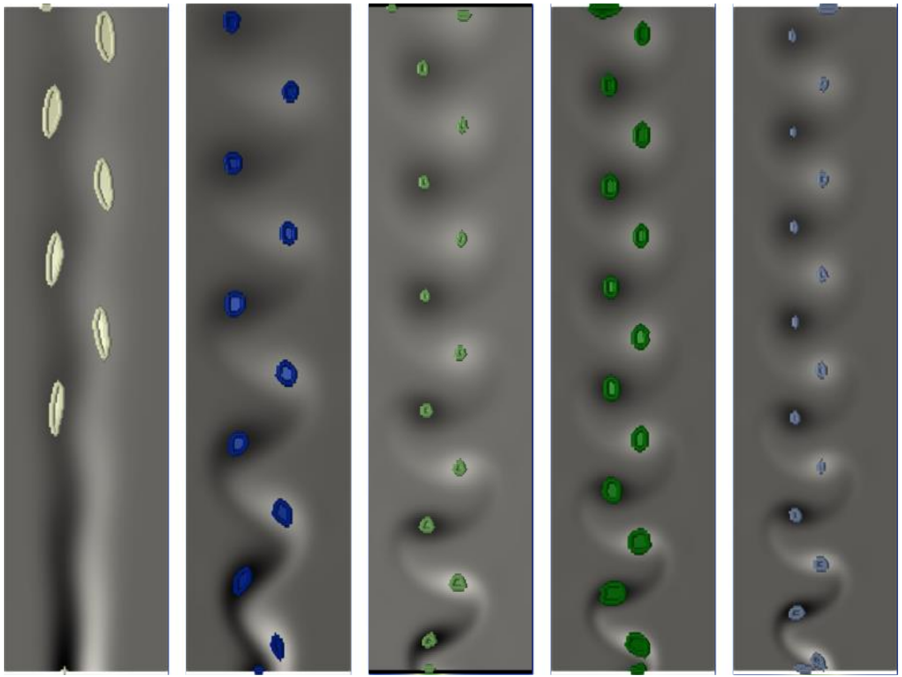
- 45 von Karman vortex streets
- Scalar data: orthogonal component of the curl
- 5 different fluids

Vortex street ensemble

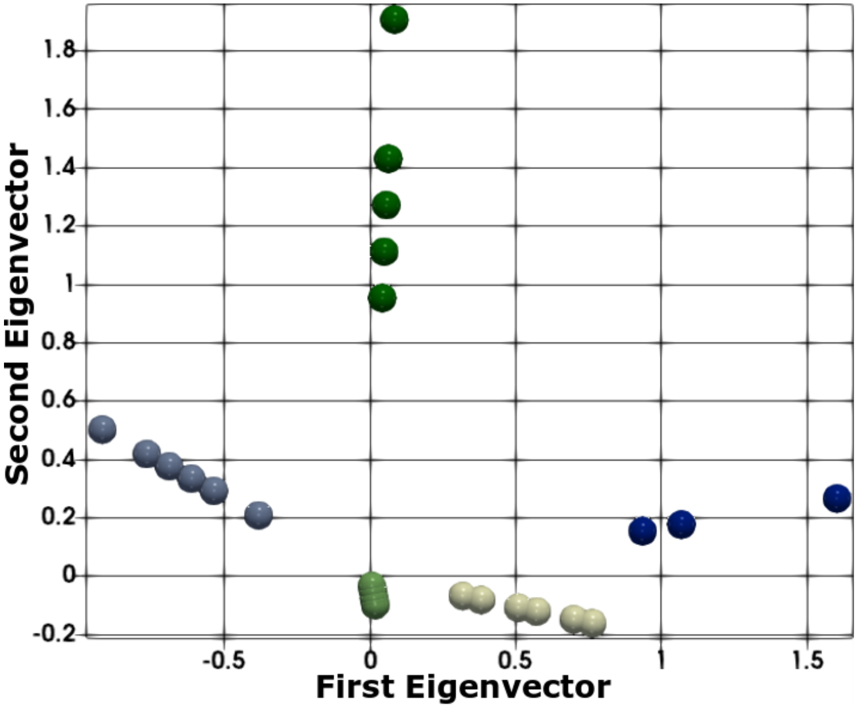


Mandatory critical points on all the ensemble members

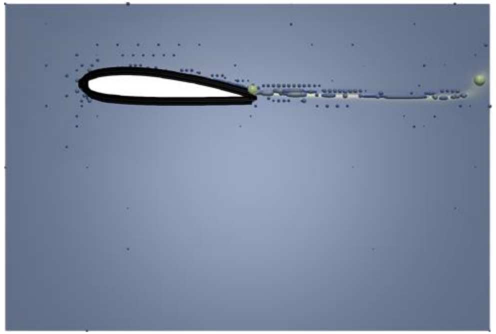
Vortex street ensemble

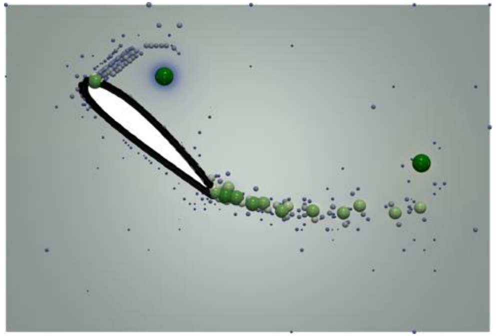


Mandatory critical points for each cluster



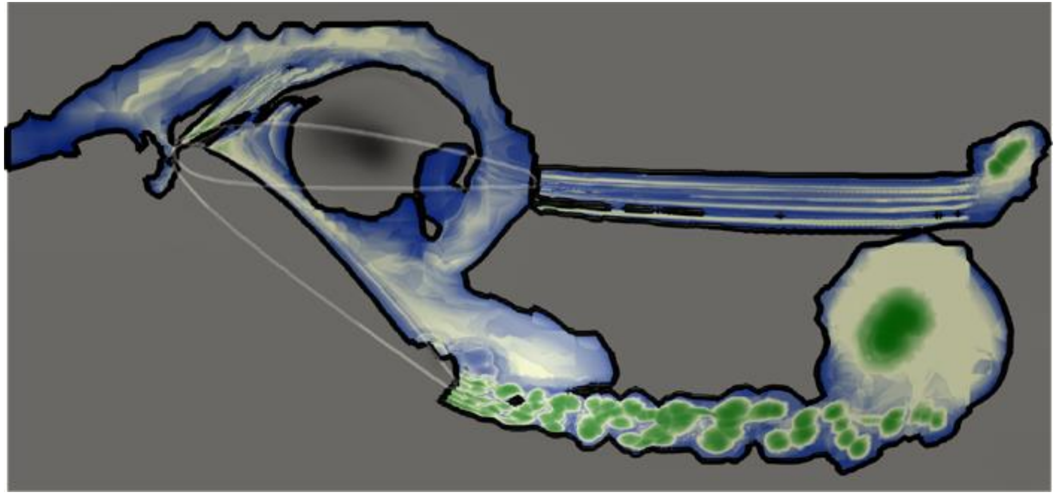
Starting vortex





12 runs of a 2D simulation

- formation of a vortex behind a wing



Mandatory critical points on all the ensemble members

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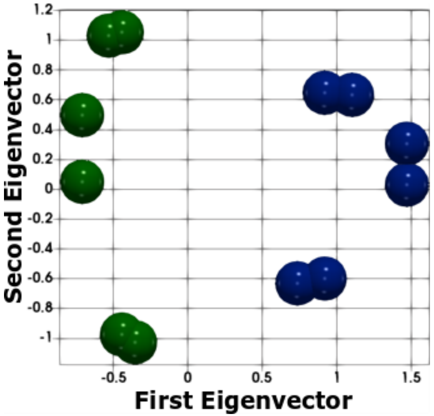

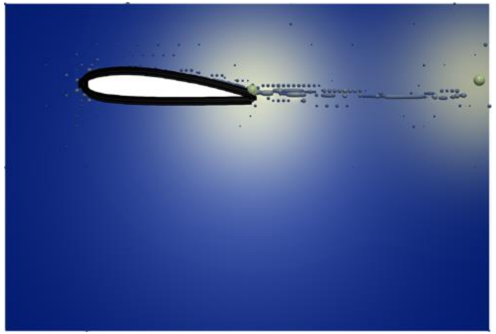

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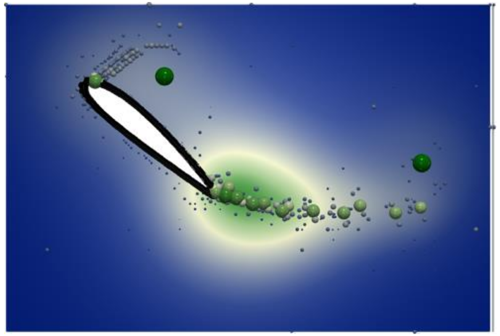
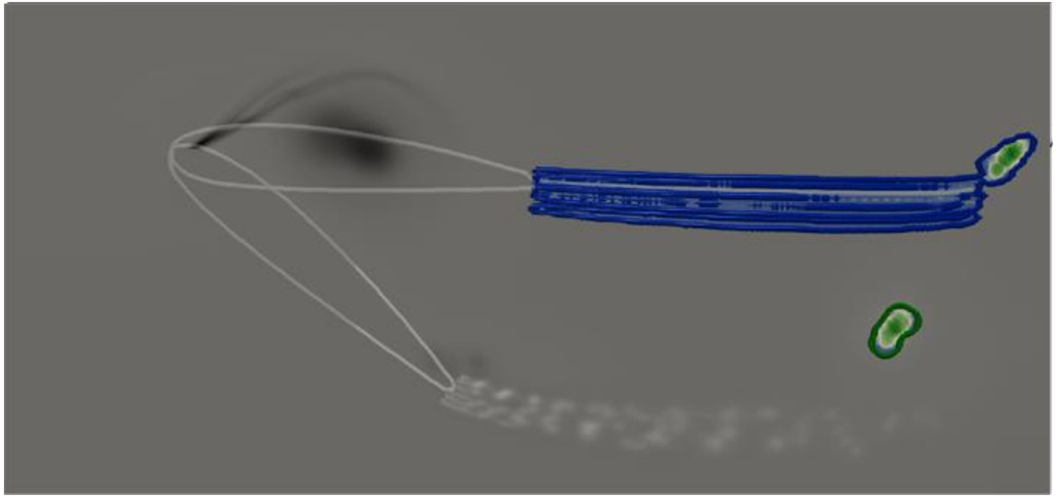
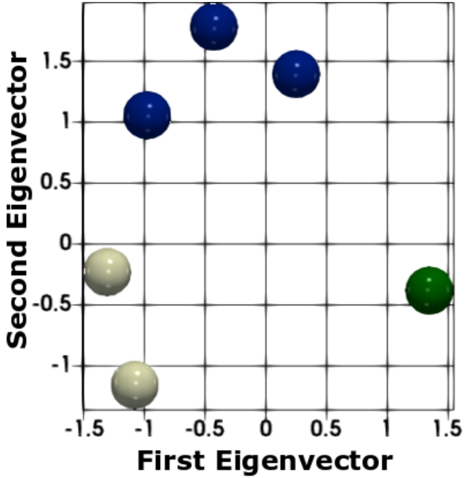


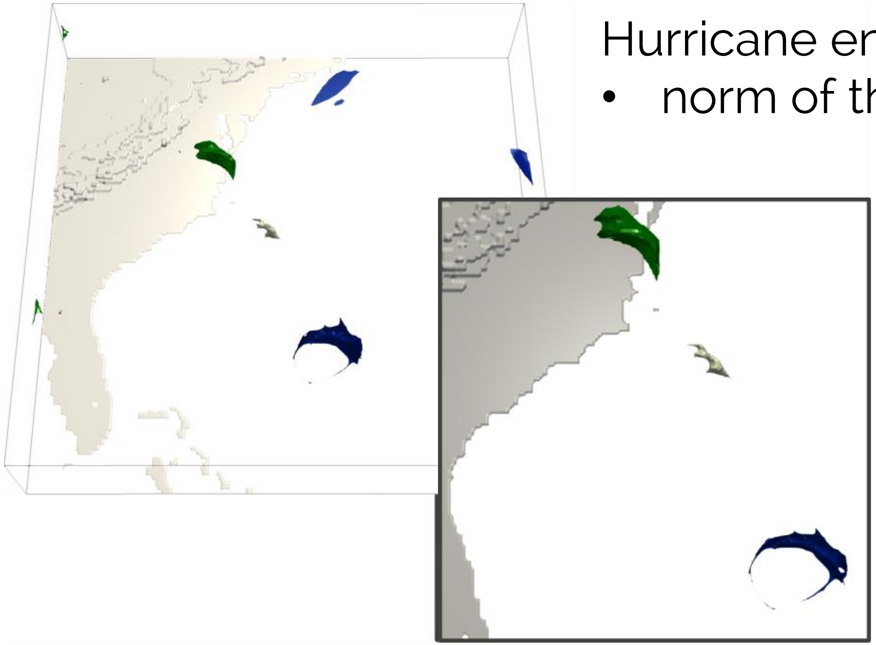


Isabel ensemble



Hurricane ensemble:

- norm of the wind velocity



3 distinct states of the hurricane (formation, drift and landfall)

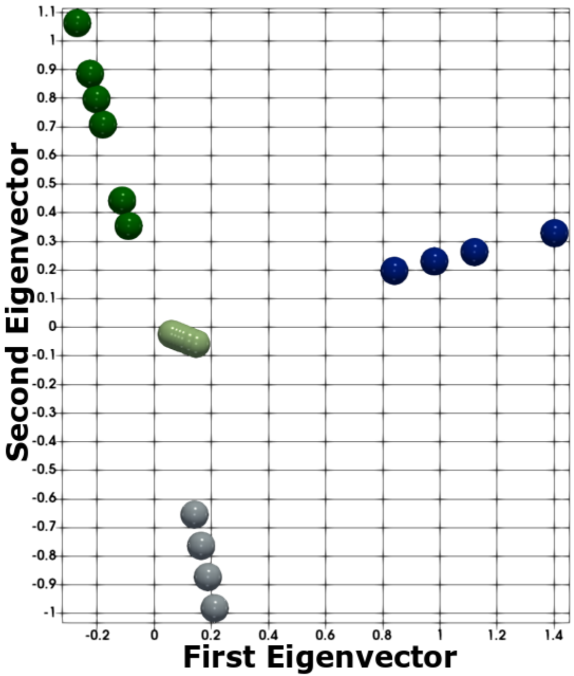
Sea surface height ensemble

48 observations:

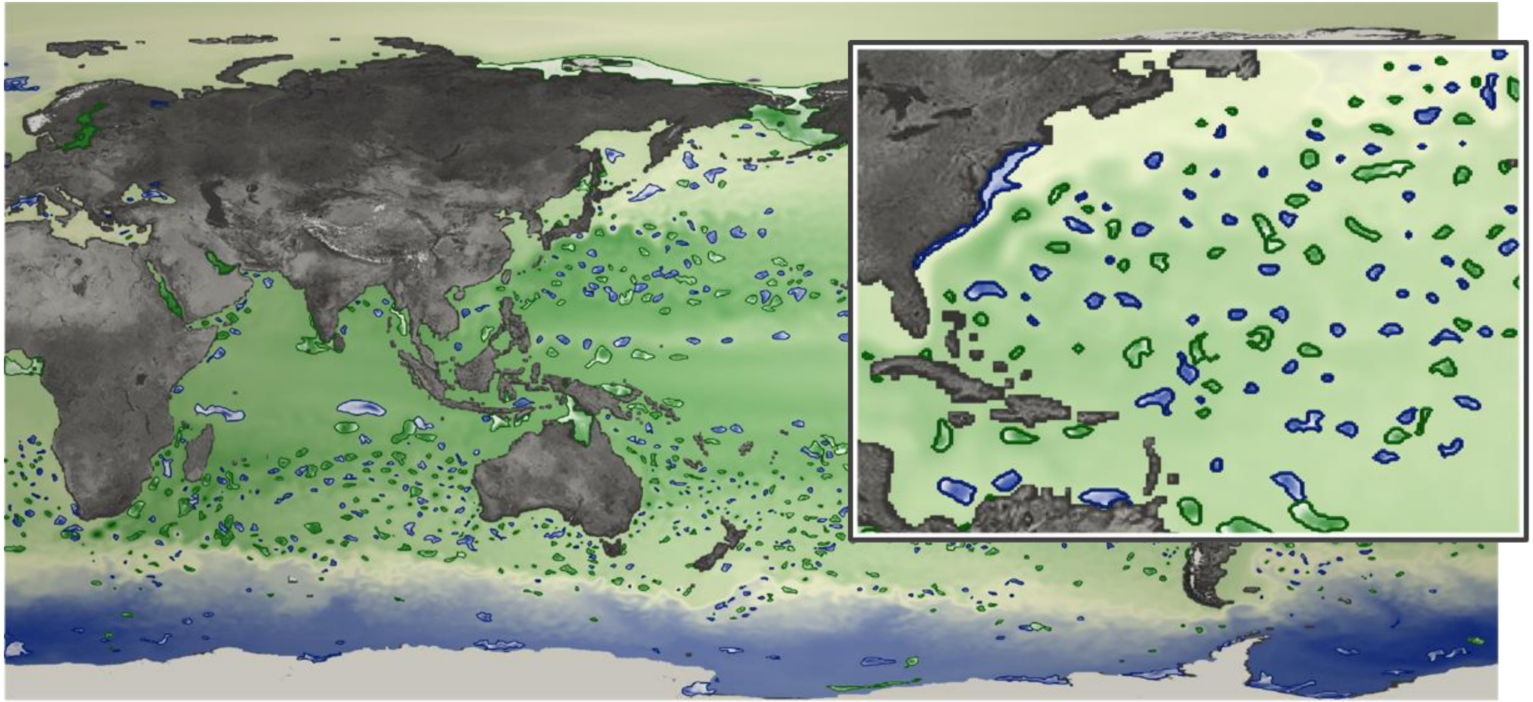
- extrema in the height at the center of eddies

Four identified clusters

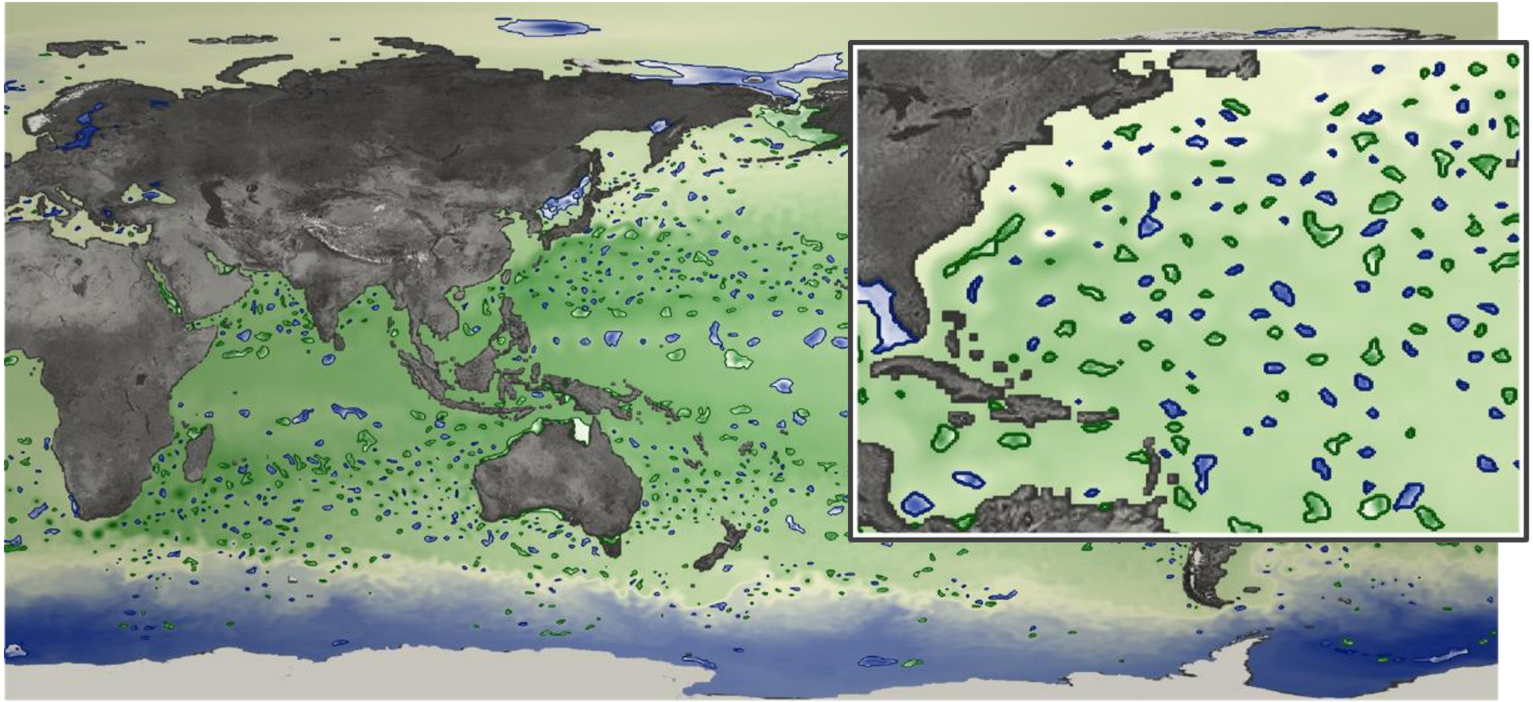
- Winter
- Spring
- Summer
- Fall



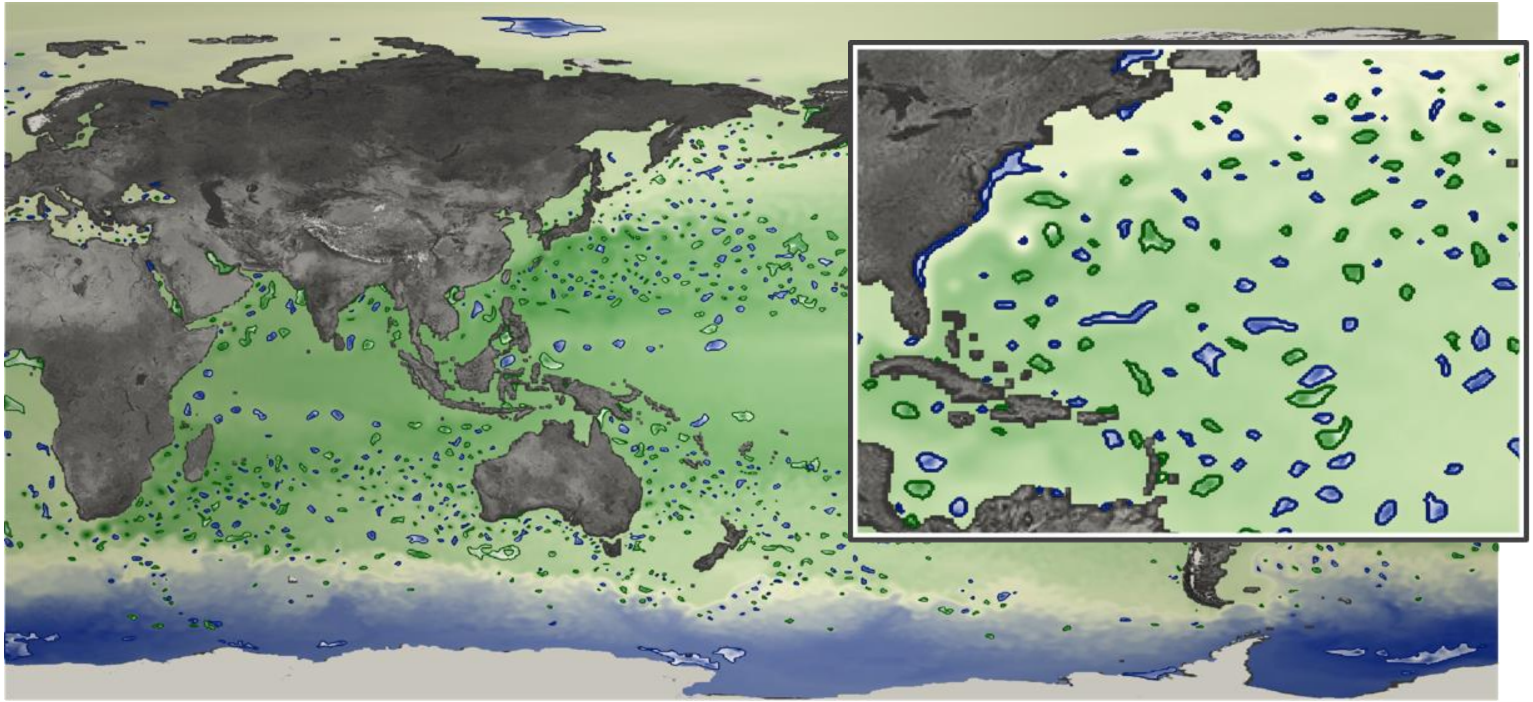
Persistence atlas: winter



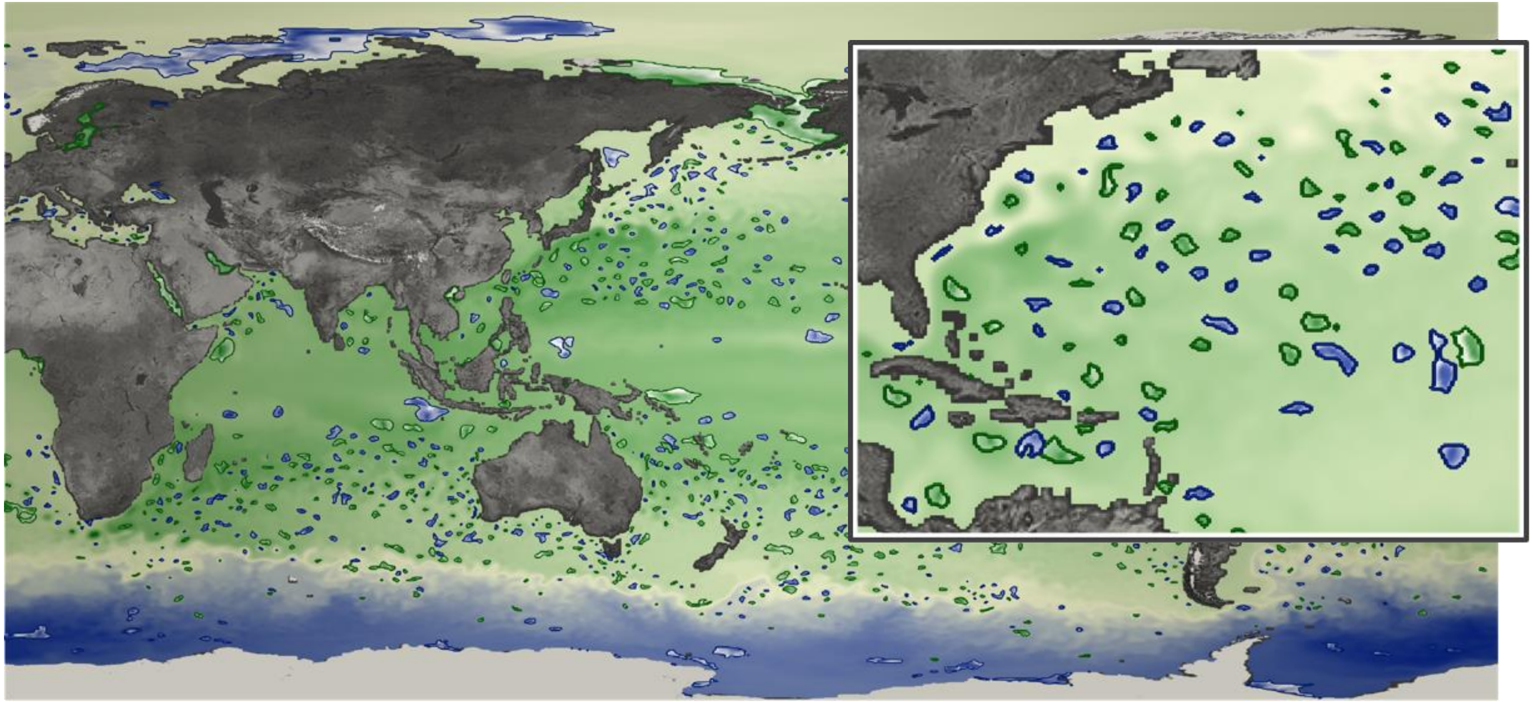
Persistence atlas: spring



Persistence atlas: summer



Persistence atlas: fall



Timings

- Persistence maps computed as a pre-process
- Interactive clustering

Dataset	n	$\lvert\mathscr{M}^0\rvert$	P.M.	D.M.	E.	C.	M.C.P.	**Total**
Gaussians (Fig. 3)	100	262,144	57.28	1.03	0.67	0.08	2.53	**61.59**
Vortex street (Fig. 1)	45	30,000	2.28	0.02	0.67	0.09	0.22	**3.28**
Starting vortex (Fig. 10)	12	1,500,000	61.44	0.09	0.65	0.07	9.08	**71.33**
Isabel (Fig. 11)	12	3,125,000	168.70	0.18	0.63	0.07	41.84	**211.68**
Sea Surface Height (Fig. 12)	48	1,036,800	290.25	0.99	0.65	0.08	8.38	**300.35**

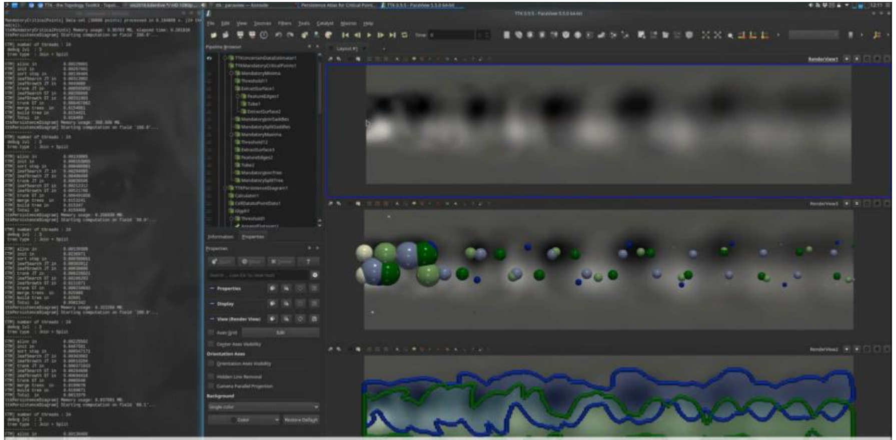
Limitations

- Persistence maps computation time
  - improvement possible
- Persistence maps do not take saddle points into account
- Cluttered visualization of the atlas
  - select subset of clusters,
  - improve strategies for the overall visualization of the atlas

Summary

- Visual analysis of the variability of critical points in ensemble data:
  - Dominant patterns found in the ensemble in term of critical point layouts
  - Local confidence regions in 3D

→Emphasis of the overall structure of the ensemble


Von Karman ensemble: curl orthogonal component, 45 runs (varying viscosity)
Top: mean field (obstacle: left)

Implementation

Topology Toolkit: https://topology-tool-kit.github.io
Scikit-Learn: http://scikit-learn.org

Persistence Atlas for Critical Point Variability in Ensembles
Guillaume Favelier, Noura Faraj, Brian Summa and Julien Tierny
IEEE Transactions on Visualization and Computer Graphics (Proc. of IEEE VIS 2018)

Ressources

Cours sur la persistent homologique pour la TDA :

Frédéric Chazal INRIA Saclay-Ile-de France

https://geometrica.saclay.inria.fr/team/Fred.Chazal/slides/Persistence2018.pdf

Tutoriaux ttk :

Clustering de points

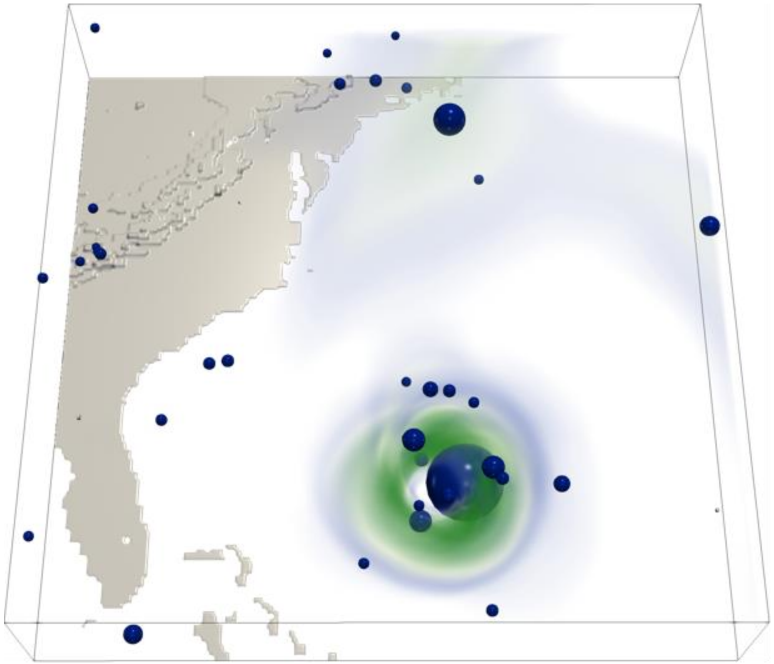
https://topology-tool-kit.github.io/tutorials.html#high

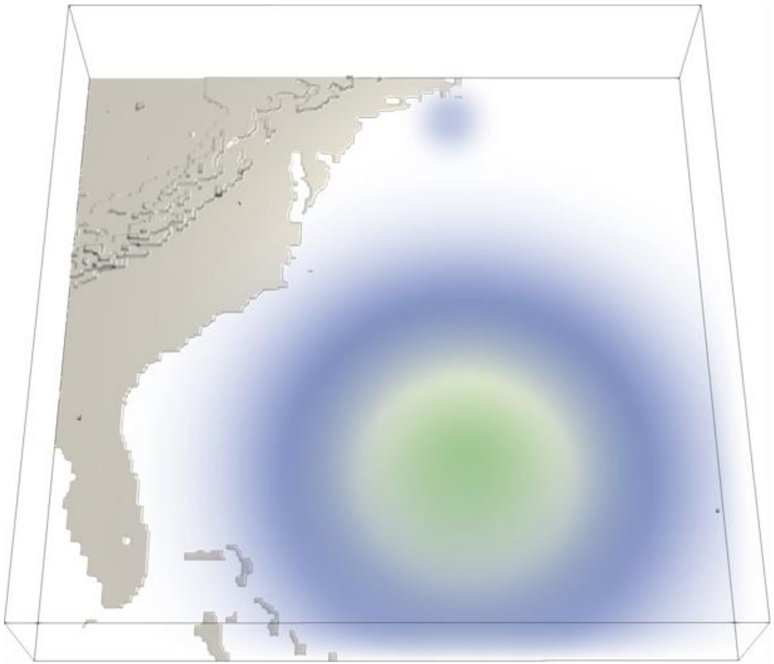
Exercices

Persistent Homology for Dummies

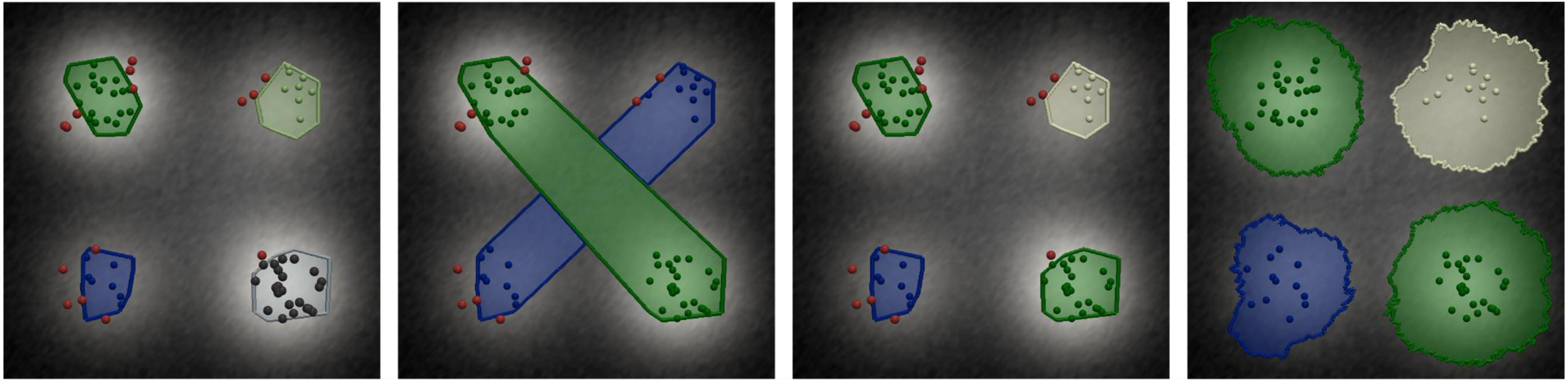
https://topology-tool-kit.github.io/persistentHomologyDummies.html

Stability





Comparison



Kernel based methods for persistence diagrams
[Reininghaus et al 2015]